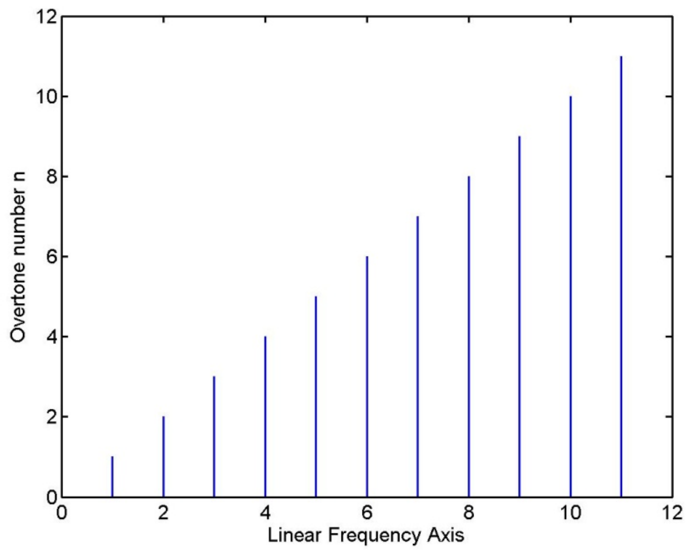
Friday 6
Intervals, overtones, and axis.

The overtone series for one-dimensional standing waves on a medium of length $\boldsymbol{L}$ and wave speed $\boldsymbol{S}$ is:

$$f_n = \frac{nS}{2L}$$

Here $\boldsymbol{n}$ is an integer: 1,2,3,4,… Each of these overtones corresponds to a "mode of vibration" of the system. Thus the sequence of mode frequencies is linearly spaced.

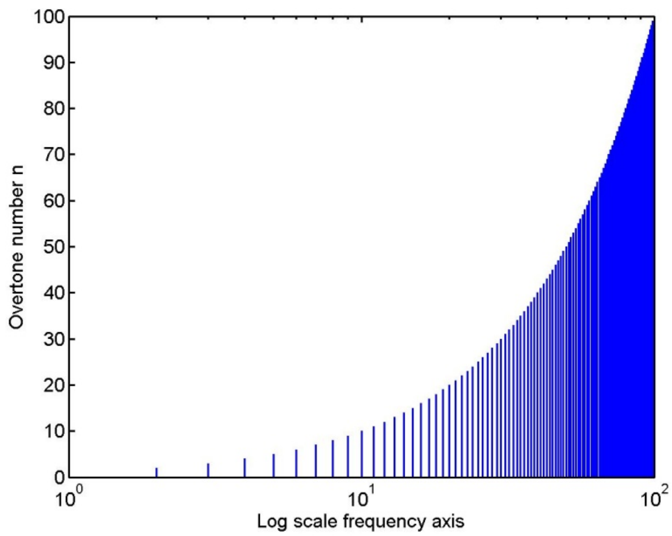


It turns out that the lowest modes tend to sound good together so there are various musical chords that result when you add together vibrations at the various mode frequencies. For example if you play modes 4, 5, and 6 together you have what is called a **major triad**. If you change the length $\boldsymbol{L}$, these modes 4, 5, and 6 will still form a major triad, but one that starts on a different pitch. This means that the musical properties depend on **ratios** of the frequencies. The major triad involves the ratios 5/4, 6/5, and 6/4. Here is a list of some musically relevant ratios (called "intervals").

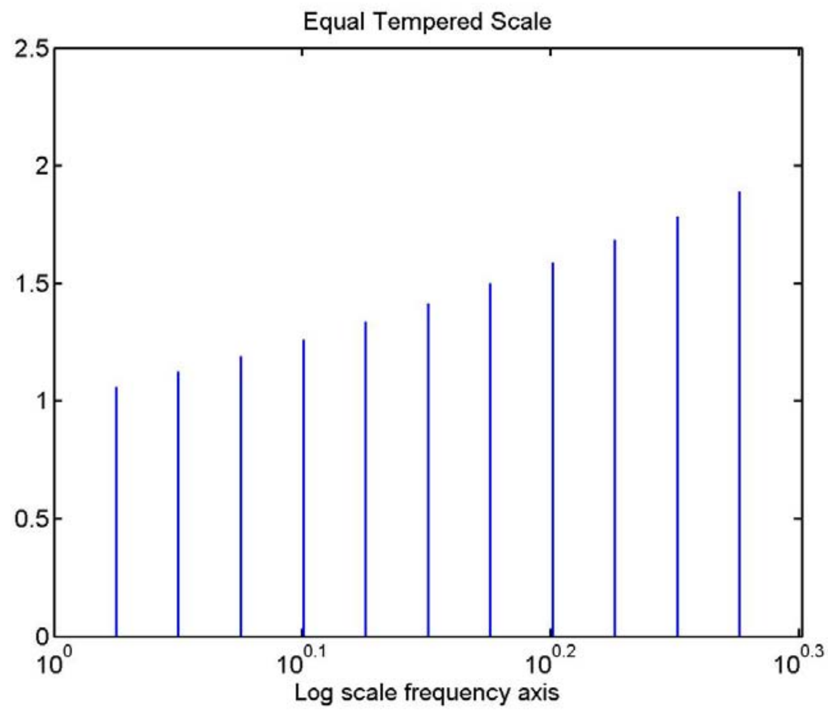
Frequency Ratio	**Name**	**Equal Temperament approx.**
2:1	Octave	2
5:3	Major sixth	1.6818
3:2	Perfect fifth	1.4983
4:3	Perfect fourth	1.3348
5:4	Major third	1.2599
6:5	Minor third	1.1892

A sequence of notes separated by constant *intervals* are equally spaced on a logarithmic axis. If you want to make a musical scale where you can play standard musical intervals starting on any note you want, then you need a scale which has equally spaced notes on a log axis. This is known as the equal tempered scale.

Firstly, if you plot the overtone series on a log axis you note that they are not equally spaced on a log axis.



The equal tempered scale is based on dividing the octave interval into twelve equal parts on a log axis.



Each step is a factor of $2^{1/12}$ higher than the one before. Since this ratio is not a rational number (cannot be written as the ratio of two whole numbers) the equal tempered scale does not allow you to play the exact musical intervals in the table (see the approximations that you can play in the third column). What this means is that there is no perfect tuning system that allows you to play perfect rational ratios starting on any note (starting on different notes corresponds to playing in a different "key").

Friday 6

Name:____________________________ Section__________

The overtone series of a hose.

By swinging a hose that is open on both ends you can excite and hear the various overtones. The open hose will have pressure nodes at both ends and some number of half-wavelengths in between. The mode frequencies include all the harmonics of the fundamental. The hose will be swung at various speeds. Make a list of the frequencies.

I will just put calculated values here.

lowest frequency	____________________	________340____
	____________________	________510____
	____________________	________680____
	____________________	________850____
	____________________	________1020___

1) Is the lowest note that we observed the fundamental frequency? How can you determine this?

No it isn't. These frequencies are in the ratio of 3:4:5:6. In other words 510=3*170, 680=4*170, 850=5*170, and 1020=6*170. This is just as we would expect for the overtone series of a pipe that is open at both ends, but we did not hear the fundamental which we can see from this series should be at 170 Hz. We were able to hear n=2, but did not get a good measurement.

2) When the tube is played like a brass instrument, what is the pitch you hear?

____________________ ______85_______

How is this pitch explained?

When you play it like a brass instrument your lips are like a closed end to the tube. Because the hose is a cylinder (and not conical like actual brass instruments), we would expect to hear only odd overtones when we close one end. The fundamental frequency for a pipe that is closed at one end, has ¼ wavelength in the tube. So once you put your lips on the end to play the note the fundamental shifts from having ½ wavelength in the tube (this would be 170 Hz) to having ¼ wavelength in the tube. The longer wavelength corresponds to lower pitch of 85.

3) If the length of the tube is about 1m and the speed of sound is 340m/s, predict the frequencies that you should hear for the various modes. Write them near the measured values.

For the tube that is open at both ends you know that some complete number of half wavelengths will be in the tube. This gives the formula.

$f_n = \frac{nv}{2L}$ The fundamental frequency here is n=1, $\frac{340m/s}{2*1m} = 170Hz$

When it is played like a brass instrument, one end is closed and you must use a different formula: $f_n = \frac{V}{4L}n$ (where n is only odd numbers 1,3,5,7) The fundamental is n=1 and is at $\frac{340m/s}{4 \bullet 1m} = 85Hz$.